Quantum Optics P572 Teaser Question Feb. 10, 2015

**Question:** *$N$-level systems with $N > 2$.*

There is a one-to-one correspondence between pure states of spin-1/2 with points on the surface of a Bloch sphere, or between mixed spin-1/2 states with points in the interior of the Bloch sphere. I can certainly construct the Bloch vector $\langle \mathbf{S} \rangle$ for spin-1 states, but the mapping is no longer one-to-one. That is, given a state, I have a unique Bloch vector, but the reverse is not true: some Bloch vectors can arise from more than one quantum spin-1 state.

*Can you argue why the Bloch vector doesn't uniquely determine the spin-1 state?* Hint: try counting the number of parameters. *Can you explicitly construct an example of two different spin-1 states with the same Bloch vector?*

Can you think of other ways to visualize the spin-1 state? There is one way, sometimes called "Dirac stars." We can view a pure spin-1 state as the projection of a product of two spin-1/2's onto the spin-1 sector (two spin-1/2's give rise to a superposition of spin-0 and spin-1 states). That is, we can associate two Bloch vectors associated with the spin-1/2's to a spin-1! *Can you draw states of spin-1 systems (using Dirac stars) that have different symmetry properties under spin rotation than any spin-1/2 state?* Hint: Can you come up with reflection symmetric Dirac stars? What about spin-1/2's?