

Question: A quantum harmonic oscillator is described by the Hamiltonian

$$H_h = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2 \quad (1)$$

with spatial coordinate r .

A quartic oscillator is defined by

$$H_q = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^4. \quad (2)$$

If I define $x = r^2$, however, the quartic oscillator Hamiltonian is simply

$$H_q = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2. \quad (3)$$

This looks the same as Eq. 1, yet the spectra of these two problems are certainly different! Where did I screw up?
The resolution of this puzzle is important for a point often glossed over when quantizing Maxwell's equations.